Money, Unit of Account, and Nominal Rigidity

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Introduction: Motivation & Objective

- Monetary economics so far has focused almost exclusively on money as a medium of exchange (MoE):
  - Since Jevons (1875), conventional wisdom has been that money should be divisible, recognizable, and portable in order to perform the role of an MoE.
  - Kiyotaki and Wright (1989); Williamson and Wright (1994); Kocherlakota (1998)
- The role of money as a unit of account (UoA) has stayed out of the spotlight in monetary economics.
  - During some hyperinflations, an object that was anchored on real value emerged as a UoA, whereas fiat money played the role of an MoE only.
  - During the German hyperinflation in the early 1920s, prices were usually quoted in terms of a gold Mark (0.358 grams of fine gold) rather than a paper Mark (fiat money).
  - More recently, in Chile, there is a CPI-indexed imaginary UoA called *Unidad de Fomento* (UF) in addition to Peso as a UoA as well as an MoE.

Digression: *Unidad de Fomento* (UF)

- The UF was introduced in 1967 by the Chilean government, though it only came into wide use as a unit-of-account in the 1980s.
- The UF is defined as the amount of currency units, Pesos, necessary for Chileans to buy a representative basket of consumer goods. The amount of pesos in one UF, or the Peso-to-UF exchange rate, is calculated daily, and is published on the Banco Central's website. The daily value is interpolated from the previous month's consumer price index.
- Real estate, rent, mortgages, car loans, long term government securities, taxes, pension payments, and alimony are all priced using UF. On the other hand, wages, consumer good prices, and stock prices are expressed in peso terms.
- Goods and services quoted in terms of the UF can only be purchased with an entirely different medium–Pesos.

Introduction: Motivation & Objective

- Considering that money is sufficiently divisible, recognizable, and portable in the modern fiat-money system, the existence of a separate UoA implies that the ideal properties of money as an MoE advocated by Jevons (1875) do not necessarily guarantee its role as a UoA.
- We investigate whether any other properties are necessary to perform the two representative roles of money as an MoE and a UoA.
  - We consider the choice of a UoA in a micro-founded model of fiat money as in Lagos and Wright (2005).
  - Motivated by the failure of money as a UoA during a hyperinflation, we incorporate the level of inflation and its volatility into the model.
  - Also, as properly pointed out by Fisher (1913), some cost is assumed to incur in converting prices quoted in a separate UoA into their MoE equivalents by which payments are made.

Introduction: Main Results

- It is not the level of inflation but its volatility that determines whether money (MoE) becomes an active UoA.
  - In the absence of inflation uncertainty, a buyer can secure a stable consumption by choosing money as a UoA as well as an MoE regardless of the conversion cost.
  - In the presence of inflation uncertainty, money can still become both an MoE and a UoA as long as the conversion cost of a separate UoA is higher than its maximum buyers are willing to bear for ensuring stable consumption.
  - If there is considerable uncertainty in inflation, money can only fulfill the role of an MoE and an object anchored on real value would replace money as an active UoA at the expense of conversion cost.
- Uncertainty on the value of fiat money relies on not the physical properties of money but the overall economic conditions such as uncertainty in money supply $\Rightarrow$ A rationale for an inflation-targeting monetary policy.

Introduction: Main Results

- The choice of a UoA in the presence of fiat money as an MoE determines endogenously the nominal price rigidity or flexibility:
  - An economy adopting an object anchored on real value as a UoA yields the nominal price flexibility in the sense that the price level varies with the growth rate of money supply.
  - In an economy adopting money as a UoA as well as an MoE, the price level is not adjusted immediately with the money growth rate, implying the short-run nominal rigidity.
- In an economy adopting money as a UoA as well as an MoE, output production is positively correlated with the money supply, which is reminiscent of a short-run Phillips curve.
- Our theory of money as a UoA sheds a new light on the issues of the nominal rigidity and the Phillips curve.

Introduction: Related Studies

- Doepke and Schneider (2013)
  - The role of money as a UoA defined as "the good in which the value of future payments is specified."
  - Inflation matters for the choice of a UoA due to its redistribution effect between borrowers and lenders.
- Kim and Lee (2013)
  - The two roles of money can be separated in the medieval commodity-money system if the likelihood of debasement and its rate are sufficiently high.
  - But modern fiat-money system differs from medieval commodity-money system, particularly in terms of divisibility and recognizability.

Model: Economic Environment

- Discrete time and a $[0, 1]$ continuum of infinitely-lived agents with discount factor $\beta \in (0, 1)$.
- In each period, there are two markets which open sequentially:
  - A decentralized market (DM) where "search good" (or the DM-good) is produced and consumed through bilateral or pairwise trade using money.
  - A competitive centralized market (CM) where "general good" (or the CM-good) is produced and consumed by all agents.
- The utility from consuming $q \in \mathbb{R}_+$ units of the DM-good is given by $u(q)$ where $u'' < 0 < u'$, $u'(0) = \infty$, and $u'(\infty) = 0$. The disutility from producing q units of the DM-good is given by q according to a linear production technology.
- An agent enjoys $v(x)$ from consuming x units of the CM-good where $v(\cdot)$ has the same properties as $u(\cdot)$. An agent suffers disutility x from producing x units of the CM-good.

Model: Economic Environment (continued)

- Money is divisible, durable, recognizable, and portable:
  - The money stock evolves over time according to $M_t = \mu_t M_{t-1}$ where μ_t is a random variable such that

$$\mu_t = \begin{cases} \mu_t^h = \bar{\mu}(1+\varepsilon) & \text{with probability} \quad \rho \\ \mu_t^l = \bar{\mu}(1-\varepsilon) & \text{with probability} \quad 1-\rho. \end{cases}$$

    where $\varepsilon \in (0,1)$ captures the (short-run) volatility of inflation and $\rho = 1/2$ so that $\mathbb{E}(\mu_t) = \bar{\mu}$ captures the (long-run) trend of inflation.
  - Assume $\mu^l > \beta$ so that the nominal interest rate is positive.

Model: Timing of Events within a Period

- At the beginning of the DM, each agent becomes
  - a buyer with probability $1/2$, who can consume the DM-good but cannot produce it;
  - a seller with probability $1/2$, who can produce the DM-good but cannot consume it.
- Each seller simultaneously and competitively posts the followings:
  - q^m units of the DM-good in exchange for a unit of money;
  - q^u units of the DM-good per unit of the CM-good.
- A submarket is formed by a set of sellers posting the same price and each buyer directs towards one that posts the most attracted terms of trade.
- In each submarket, buyers and sellers are randomly matched according to $\alpha = \min\{1, \lambda\}$ where λ denotes the ratio of sellers to buyers in the submarket.

Model: Timing of Events within a Period (continued)

- A buyer chooses a UoA–whether to trade according to the price quoted in terms of money (q^m) or the price quoted in terms of the CM-good (q^u).
- Money growth shock μ_t is realized and each agent receives a lump-sum transfer of money.
- A matched pair of buyer and seller trade according to the pre-determined terms of trade. If a buyer chooses the price quoted in the CM-good, the value of the CM-good should be converted into its money equivalents at some disutility cost κ before payments are made.

Stationary Equilibrium

Focus on a stationary equilibrium in which the end-of-period real money balance is constant over time:

$$\phi_{t-1} M_{t-1} = \phi_t^h \mu_t^h M_{t-1} = \phi_t^l \mu_t^l M_{t-1}$$

where ϕ^i for $i \in \{h, l\}$ is the real price of money in terms of the CM-good when the realized money growth rate is μ^i.

Stationary Equilibrium: Money Demand

In the CM, agents produce, consume the CM-good, and choose the balance of money to be carried into the next-period DM. The problem for an agent entering the CM with a monetary wealth m is

$$W(m) = \max_{(x,y,m_{+1})} [v(x) - y + \beta V_{+1}(m_{+1})]$$

$$\text{s.t. } x + \phi m_{+1} = y + \phi m$$

where m_{+1} denotes the demand for money to be carried into the DM next period and $V_{+1}(m_{+1})$ is given by

$$V_{+1}(m_{+1}) = \frac{1}{2}\left[V^b_{+1}(m_{+1}) + V^s_{+1}(m_{+1})\right]$$

with $V^b(m)$ and $V^s(m)$ denoting the value function for a buyer and a seller in the DM, respectively.

Stationary Equilibrium: Money Demand (continued)

Substituting y from the constraint into $W(m)$,

$$W(m) = \phi m + \max_{x} \{v(x) - x\} + \max_{m_{+1}} \{\beta V_{+1}(m_{+1}) - \phi m_{+1}\}.$$

- $v'(x) = 1$, which implies that all agents consume x^* units of the CM-good such that $x^* = \arg\max[v(x) - x]$ regardless of m.
- $\phi = \beta V'_{+1}(m_{+1})$, implying no wealth effect and hence all agents exit the CM with an identical balance of money $m_{+1} \Rightarrow$ A degenerate distribution of money holdings at the beginning of each period.
- The envelope condition, $W'(m) = \phi$, implies that the value function $W(m)$ is linear as in the typical Lagos-Wright model with a quasi-linear utility function.

Stationary Equilibrium: Unit-of-Account Choice

In the DM, a seller posts prices (q^m, q^u) and the value function for a seller with m units of money should satisfy

$$V^s(m) = \max_{(q^m,q^u)} \left\{ \left(\frac{\alpha}{2}\right) \sum_{i\in\{h,l\}} \left[\left(\hat{z}^i - \hat{m}^i q^m\right) + \left(\tilde{z}^i - \tilde{z}^i q^u\right) \right] + W(m+\tau) \right\}$$

where $\hat{z}^l = \phi^l \hat{m}^l$, $\hat{z}^h = \phi^h \hat{m}^h$, $\tilde{z}^l = \phi^l \tilde{m}^l$, $\tilde{z}^h = \phi^h \tilde{m}^h$, and $\hat{m}$ $(\tilde{m})$ denotes the nominal amount of transaction when the trade is made according to the price posted in terms of money (CM-good), and τ is a lump-sum transfer of new money such that

$$\tau = \begin{cases} \tau^h = (\mu^h - 1)M_{-1} & \text{if } \mu = \mu^h \\ \tau^l = (\mu^l - 1)M_{-1} & \text{if } \mu = \mu^l \end{cases}$$

Stationary Equilibrium: Unit-of-Account Choice (cont'd)

A buyer spends all the money she holds and hence, from $V^s(m)$,

$$(m+\tau^i)q^m \leq (m+\tau^i)\phi^i$$

$$(m+\tau^i)q^u \leq (m+\tau^i)$$

for $i \in \{h, l\}$. Otherwise it is cheaper for a seller to acquire money in the following CM and hence she is not willing to trade in the DM.

- $\phi^h < \phi^l$ from the constant real balances $(m+\tau^h)\phi^h = (m+\tau^l)\phi^l$ and $(m+\tau^h) > (m+\tau^l)$, q^m should satisfy $q^m \leq \phi^h < \phi^l$ $\Rightarrow q^m = \phi^h$ because the competition among sellers drives equilibrium profit to zero.
- $q^u \leq 1 \Rightarrow q^u = 1$ due to the zero-profit equilibrium condition for sellers.

Stationary Equilibrium: Unit-of-Account Choice (cont'd)

For the posted prices (q^m, q^u), the value function for a buyer with m units of money in the DM should satisfy

$$V^b(m) = \max_{(\mathbb{I},\hat{m},\tilde{m})} \left\{ \begin{array}{c} \left(\frac{1-\mathbb{I}}{2}\right)\alpha \sum_{i\in\{h,l\}} \left[u(\hat{m}^i q^m) - \hat{z}^i\right] + \\ \left(\frac{\mathbb{I}}{2}\right)\alpha \sum_{i\in\{h,l\}} \left[u(\tilde{z}^i q^u) - (\tilde{z}^i + \kappa)\right] \end{array} \right\} + W(m+\tau)$$

subject to $\hat{m}^i \leq (m+\tau^i)$ and $\tilde{m}^i \leq (m+\tau^i)$ for $i \in \{h,l\}$.

- A measure of sellers is a half and a measure of buyers is also a half because all buyers are willing to visit the single submarket formed by identical and competitive sellers $\Rightarrow \alpha = 1$ in $V^b(m)$:

$$V^b(m) = \max_{\mathbb{I}} \left\{ \left(\frac{1-\mathbb{I}}{2}\right) \sum_{i\in\{h,l\}} u(\boldsymbol{m}^i q^m) + \mathbb{I} \times \left[u(\boldsymbol{z} q^u) - \kappa\right] \right\} + W(0)$$

- For money posting ($\mathbb{I} = 0$), the quantity of the DM-good produced and consumed relies on the realized inflation via $\boldsymbol{m}^i = m + \tau^i$, whereas it is irrelevant for the CM-good posting ($\mathbb{I} = 1$) since $\boldsymbol{z} = (m+\tau^l)\phi^l = (m+\tau^h)\phi^h$.

Stationary Equilibrium: Active Unit of Account

- A buyer's payoff from money posting ($\mathbb{I} = 0$):

$$\begin{aligned} S_m &= \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) - \phi^h \boldsymbol{m}^h\right] + \frac{1}{2}\left[u(\boldsymbol{m}^l q^m) - \phi^l \boldsymbol{m}^l\right] \\ &= \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)\right] - \boldsymbol{z} \end{aligned}$$

- A buyer's payoff from the CM-good posting ($\mathbb{I} = 1$):

$$S_g = u(\boldsymbol{z} q^u) - \boldsymbol{z} - \kappa.$$

- A buyer's relative payoff from money posting:

$$\Delta = \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)\right] - \left[u(\boldsymbol{z} q^u) - \kappa\right].$$

Stationary Equilibrium: Active Unit of Account (cont'd)

Lemma

If $\kappa = 0$, $\Delta = S_m - S_g < 0$ and hence buyers always prefer the price quoted in the CM-good to the price quoted in money.

- In the absence of any cost of converting the CM-good price into money price ($\kappa = 0$), a buyer can secure a stable consumption against inflation uncertainty by choosing the price quoted in terms of the CM-good.
- This result implies that if there is some inconvenience in converting the CM-good price into money price (i.e., $\kappa > 0$), a nominal-value anchoring economy can arise where money becomes an active UoA as well as an MoE.

Stationary Equilibrium: Active Unit of Account (cont'd)

Proposition

There exists $\bar{\kappa} \in \mathbb{R}_{++}$ *such that* $\Delta|_{\kappa=\bar{\kappa}} = 0$. *For* $\kappa > \bar{\kappa}$, *buyers choose money as an active unit of account.*

- The threshold value of the conversion cost, $\bar{\kappa}$, can be interpreted as a risk premium in the sense that it captures the maximum disutility cost buyers are willing to bear for ensuring a stable consumption against inflation uncertainty.
- If the cost of converting the CM-good price into money price exceeds $\bar{\kappa}$, then agents are not willing to hedge the consumption volatility against inflation uncertainty.
- Fisher (1913) "laborious calculation in translating from the MoE into the standard of deferred payment, and back again"

Stationary Equilibrium: Active Unit of Account (cont'd)

Proposition

If there is no uncertainty in inflation ($\varepsilon = 0$), then $\bar{\kappa} = 0$ and hence money is always an active unit of account for $\kappa > 0$. In addition, $\bar{\kappa}$ increases with the uncertainty in inflation.

- If there is no uncertainty in inflation, money is an active UoA as well as an MoE even if the conversion cost is sufficiently small.
- As the volatility of inflation increases for a given conversion cost, the threshold level $\bar{\kappa}$ increases and the relative payoff from choosing money as a UoA eventually becomes negative.
- It is not the level of inflation but its volatility that matters for the choice of a UoA:
  - Even though money is ideal as an MoE (i.e., divisible, durable, recognizable and portable), it can hardly function as a UoA if its value is too unstable.
  - Despite the importance of portability as an MoE during a hyperinflation, it is irrelevant to the choice of a UoA.

Stationary Equilibrium: Active Unit of Account (cont'd)

Proposition

If there is no uncertainty in inflation ($\varepsilon = 0$), then $\bar{\kappa} = 0$ and hence money is always an active unit of account for $\kappa > 0$. In addition, $\bar{\kappa}$ increases with the uncertainty in inflation.

- This finding is consistent with Keynes (1923) who claim that as the volatility of an MoE increases, its quality as a UoA is deteriorated and an alternative UoA anchored on real value would take up the role of a UoA.
- This provides an explanation for why some countries adopted a UoA different from an MoE during hyperinflations at some inconvenience.
  - German hyperinflation in the early 1920s: gold Marks vs paper Marks
  - Some Latin American countries such as Brazil, Chile, Colombia, Ecuador, Mexico, and Uruguay which introduced a separate UoA after high inflation (Shiller 2002).

Nominal Rigidity and Non-neutrality of Money

Let p_m and p_g denote respectively the equilibrium price level (i.e., the units of money exchanged for the quantity of the DM-good) when money and the CM-good are active UoA:

$$p_m = \begin{cases} \frac{\boldsymbol{m}^h}{\boldsymbol{m}^h q^m} = \frac{1}{q^m} = \frac{1}{\phi^h} & \text{if } \mu = \mu^h \\ \frac{\boldsymbol{m}^l}{\boldsymbol{m}^l q^m} = \frac{1}{q^m} = \frac{1}{\phi^h} & \text{if } \mu = \mu^l. \end{cases}$$

$$p_g = \begin{cases} \frac{\boldsymbol{m}^h}{\boldsymbol{z} q^u} = \frac{1}{\phi^h q^u} = \frac{1}{\phi^h} & \text{if } \mu = \mu^h \\ \frac{\boldsymbol{m}^l}{\boldsymbol{z} q^u} = \frac{1}{\phi^l q^u} = \frac{1}{\phi^l} & \text{if } \mu = \mu^l \end{cases}$$

- An economy adopting money as a UoA yields the short-run nominal price rigidity, whereas an economy adopting an object that is anchored on real value as a UoA yields the nominal price flexibility.
- This has an important testable implication for the nominal rigidity: the more (fewer) firms post prices in terms of a UoA different from money, the more (less) flexible the price level becomes.

Nominal Rigidity and Non-neutrality of Money (cont'd)

- When the money is an active UoA, the quantity of the DM-good produced is positively correlated with the growth rate of money, implying a traditional short-run Phillips curve relationship.
- When the CM-good is an active UoA, the quantity of the DM-good produced is irrelevant to the realized money growth rate.
- This is consistent with Shiller (2002) who claims that if an MoE is separated from a UoA that is indexed to consumer price index, the effects of sticky prices on the macroeconomy would be substantially lessened.

Concluding Remarks

- We show that a separation of UoA from MoE arises if there is considerable uncertainty in inflation. This implies that, in order to fulfill the roles of money as a UoA as well as an MoE, stability is required other than the properties advocated by Jevons (1875): i.e., money should be divisible, durable, recognizable, portable and stable.
- The choice of a UoA in the presence of fiat money as an MoE determines endogenously the nominal price rigidity or flexibility.
  - In an economy adopting money as a UoA as well as an MoE, the price level is sticky in the sense that it is not adjusted immediately with the growth rate of money supply.
  - An economy adopting an object anchored on real value as a UoA yields the nominal price flexibility.
  - In Chile where prices of some goods are posted in *Unidad de Fomento* (CPI-Indexed-UoA) while prices of other goods are posted in Peso (MoE), it would be interesting to see whether there is any difference in the price rigidities between the two groups of goods.