Money, Unit of Account, and Nominal Rigidity

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Abstract

In order to characterize the properties required to fulfill the roles of money as a unit of account (UoA) as well as a medium of exchange (MoE), we consider the choice of a UoA in the context of a micro-founded model where inflation uncertainty exists and some conversion cost is incurred in using a UoA that is different from an MoE. We show that it is not the level of inflation but its volatility that matters for the choice of a UoA. In the presence of inflation uncertainty, money can still become both an MoE and a UoA as long as the conversion cost is higher than its maximum buyers are willing to bear for ensuring stable consumption against inflation uncertainty. Also, the choice of a UoA in the presence of fiat money as an MoE determines endogenously the nominal price rigidity or flexibility. An economy adopting money as a UoA yields the short-run nominal rigidity and the Phillips-curve relationship.

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1. Introduction

Money typically functions as a medium of exchange (MoE) and a unit of account (UoA). The former captures the role of money that facilitates trades in an environment where people cannot commit to each other or there is no record-keeping technology. The latter captures the role of money that is used to quote prices or terms of trade.

Monetary economics so far has focused almost exclusively on money as an MoE. Since Jevons (1875), conventional wisdom has been that money should be divisible, recognizable and portable in order to perform the role of an MoE.[1] Also, since Kiyotaki and Wright (1989), search-theoretic monetary models embedding the double-coincidence problem have been used widely to establish the essentiality of money. In particular, Kocherlakota (1998) shows that an intrinsically useless tangible object (i.e., fiat money) is memory in the sense that it can serve as the monitoring and record-keeping technology. That is, money enables trades between anonymous agents even in the absence of full commitment.

On the other hand, the role of money as a UoA has stayed out of the limelight in monetary economics. It appears to take for granted that prices are quoted in terms of an MoE and hence a UoA is equivalent to an MoE. However, some observations suggest that it would not be the case, particularly during a hyperinflation where both the level and the volatility of inflation were typically high. For instance, during the German hyperinflation in the early 1920s, prices were usually quoted in terms of a gold Mark (0.358 grams of fine gold) rather than a paper Mark (fiat money). Similar instances were observed more recently in some Latin American countries like Chile where, in addition to Peso as a UoA as well as an MoE, there is a CPI-Indexed imaginary UoA

[1] See Nosal and Rocheteau (2011, pp. 99-125) for more extensive survey and discussion on the implications of each property.

called *Unidad de Fomento*. That is, in some hyperinflation episodes, an object that was anchored on real value emerged as a UoA, whereas fiat money played the role of an MoE only. Considering that money is sufficiently divisible, recognizable and portable in the modern fiat-money system, the existence of a separate UoA implies that the ideal properties of money as an MoE advocated by Jevons (1875) do not necessarily guarantee its role as a UoA.

It is then quite natural to ask what additional properties are required to fulfill the roles of money as a UoA as well as an MoE. However, to the best of our knowledge, there is only a small literature that has explored thoroughly the properties of money as a UoA.[2] In this paper, we attempt to investigate whether any other properties besides those required as an MoE are necessary to perform the two representative roles of money as an MoE and a UoA successfully. Specifically, we consider the choice of a UoA in the context of a micro-founded model of fiat money as in Lagos and Wright (2005). Motivated by the failure of money as a UoA during a hyperinflation, we incorporate the level of inflation and its volatility into the model. Also, as properly pointed out by Fisher (1913), some cost is assumed to incur in converting prices quoted in terms of a separate UoA (e.g., *Unidad de Fomento*) into their MoE equivalents (e.g., Pesos) by which payments are made.

The main results are as follows. First, it is not the level of inflation but its volatility that matters for the choice of a UoA. If there is no uncertainty in inflation, money can play the roles of a UoA as well as an MoE even if the conversion cost of an imaginary

[2] Doepke and Schneider (2013) study the role of money as a UoA that is defined as "the good in which the value of future payments is specified." In their model, inflation matters for the choice of a UoA due to its redistribution effect between borrowers and lenders. In our model, as we will see, inflation matters for the choice of a UoA due to its effect on output volatility. Kim and Lee (2013) address the question of why a UoA was separated from an MoE in the medieval commodity-money system. They show that the two roles of money can be separated if the likelihood of debasement and its rate are high enough. But modern fiat-money system differs essentially from medieval commodity-money system, particularly in terms of divisibility and recognizability.

UoA is close to zero. In the absence of inflation uncertainty, a buyer can secure a stable consumption by choosing money as a UoA as well as an MoE regardless of the conversion cost. However, as inflation uncertainty increases, money is preferred as a UoA only if the conversion cost is sufficiently high. In the presence of inflation uncertainty, money can still become both an MoE and a UoA as long as the conversion cost of a separate UoA is higher than its maximum buyers are willing to bear for ensuring stable consumption against inflation uncertainty. If there is considerable uncertainty in inflation, money can only fulfill the role of an MoE and fails to serve as a UoA. That is, if the value of money as an MoE is quite unstable, an object anchored on real value would replace money as an active UoA at the expense of conversion cost. This provides an explanation for the separation of a UoA from an MoE during the hyperinflations in Germany and in some Latin American countries.

Noting that fiat money is an intrinsically useless object, uncertainty on the value of fiat money would crucially rely on not the physical properties of money but the overall economic conditions such as uncertainty in money supply. Our results then provide another rationale for an inflation-targeting monetary policy which essentially aims at reducing inflation uncertainty by stabilizing inflation around its target rate.

Second, the choice of a UoA in the presence of fiat money as an MoE determines endogenously the nominal price rigidity or flexibility. An economy adopting an object anchored on real value as a UoA yields the nominal price flexibility in the sense that the price level varies with the growth rate of money supply. In an economy adopting money as a UoA as well as an MoE, however, the price level is not adjusted immediately with the money growth rate, implying the short-run nominal rigidity. Also, output production is positively correlated with the money supply, which is reminiscent of a short-run Phillips curve. Noting that money plays the roles of an MoE as well as a

UoA in most modern economies, our theory of money as a UoA sheds a new light on the issues of the nominal rigidity and the Phillips curve.

The paper is organized as follows. Section 2 describes the model economy, followed by the equilibrium characterization in Section 3. Section 4 presents the choice of an active UoA in equilibrium. Section 5 discusses the implications for nominal price rigidity. Section 6 summarizes the paper with a few concluding remarks.

2. Model

The background environment is a version of Lagos and Wright (2005). Time is discrete and there is a $[0, 1]$ continuum of infinitely-lived agents. In each period, there are two markets, a decentralized market (hereinafter "DM") and a centralized market (hereinafter "CM"), which open sequentially. There are two perishable and perfectly divisible consumption goods, DM-good and CM-good. The DM-good is produced and consumed following bilateral trade in the decentralized market, whereas the CM-good is produced and consumed in the centralized competitive market.

At the beginning of the DM, each agent receives one of the two equally probable preference shocks; with probability one half an agent can consume the DM-good but cannot produce the DM-good (i.e., a buyer) and with the remaining probability an agent can produce the DM-good but cannot consume the DM-good (i.e., a seller). The utility from consuming q units of the DM-good is given by $u(q)$ where $u'' < 0 < u'$, $u'(0) = \infty$, and $u'(\infty) = 0$. The disutility from producing $q \in \mathbb{R}_+$ units of the DM-good is given by q according to a linear production technology. In the CM, all agents can consume and produce the CM-good. An agent enjoys $v(q)$ from consuming $q \in \mathbb{R}_+$ units of the CM-good where $v(\cdot)$ has the same properties as $u(\cdot)$ mentioned above. As in the DM, an agent suffers disutility x from producing $x \in \mathbb{R}_+$ units of the CM-good.

There is another object called money which is perfectly divisible, durable and intrinsically useless. For simplicity, there is no counterfeiting technology and no carrying cost of money. Hence, our money satisfies all the ideal properties advocated by Jevons (1875): i.e., money is divisible, durable, recognizable and portable. Let M_t denote the quantity of money in the period-t DM where new money is injected by lump-sum transfers. That is, the money stock evolves over time according to $M_t = \mu_t M_{t-1}$ where μ_t is a random variable such that

$$\mu_t = \begin{cases} \mu_t^h = \bar{\mu}(1+\varepsilon) & \text{with probability} \quad \rho \\ \mu_t^l = \bar{\mu}(1-\varepsilon) & \text{with probability} \quad 1-\rho. \end{cases}$$

We here assume $\varepsilon \in (0,1)$ and $\rho = 1/2$ so that $\mathbb{E}(\mu_t) = \bar{\mu}$. Notice that ε captures the (short-run) volatility of inflation, while $\bar{\mu}$ captures the (long-run) trend of inflation. We also assume $\mu^l > \beta$ to ensure an interior solution for money demand.

After the realization of the preference shock in the DM, each seller simultaneously and competitively posts prices of the DM-good in terms of money and the CM-good, respectively. The former price essentially anchors on nominal value of money, whereas the latter anchors on real value of the CM-good. Specifically, a seller commits to the terms of trade such that she will produce q^m units of the DM-good in exchange for a unit of money or q^u units of the DM-good per unit of the CM-good. A submarket is then formed by a set of sellers posting the same price. Upon observing the posted prices of all submarkets, each buyer directs towards a submarket that posts the most attracted terms of trade. In each submarket, buyers and sellers are randomly matched according to the matching function $\alpha = \min\{1, \lambda\}$ where λ denotes the ratio of sellers to buyers in the submarket. A buyer then chooses a UoA–whether to trade according to the price quoted in terms of money or the price quoted in terms of the CM-good.

Immediately after the choice of a UoA, money growth shock μ_t is realized and then a matched pair of buyer and seller trade according to the pre-determined terms of trade.[3] It is worthwhile to note that in a bilateral trade, agents cannot make any binding commitments across the DM-CM markets and their trading histories are private. Thus, all trades should be *quid pro quo* and money is essential (see, for instance, Kocherlakota 1998, Wallace 2001, Corbae, Temzelideset and Wright 2003, and Aliprantis, Camera and Puzello 2007). That is, transfer of tangible money as an MoE in exchange for the DM goods produced should be made on the spot. The CM-good can play the role of a UoA but cannot be used as an MoE because it is not available in the DM in the sense that it is produced only in the CM and cannot be stored across markets. Therefore, if a buyer chooses the price quoted in the CM-good, the value of the CM-good should be converted into its equivalent in money at some disutility cost. As pointed out by Fisher (1913), if an MoE is different from a UoA, there should be some disutility cost κ due to the laborious calculations in translating from a UoA into an MoE. We assume that κ is borne by a buyer who chooses whether to trade according to the price quoted in the CM-good or in money.

An agent seeks to maximize her expected lifetime utility given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(q_{t,b}) - q_{t,s} - \kappa \mathbb{I}_t + v(x_t) - y_t \right\} \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, $q_{t,b}$ ($q_{t,s}$) denotes the DM-good consumption (production), x_t (y_t) is the CM-good consumption (production), and $\mathbb{I}$ is an indicator taking value 1 if an agent as a buyer in the DM chooses the price quoted in terms of

[3]Money is typically inessential if agents can make commitment. But the within-market commitment required here for a price-posting equilibrium is not sufficient to render money inessential (for more formal discussion, see Lagos and Rocheteau 2005).

the CM-good (hereinafter "CM-good posting") and 0 if she chooses the price quoted in terms of money (hereinafter "money posting").

In short, the sequence of events within a period is summarized as follows. First, the preference shock (either a buyer or a seller in the DM) is realized. Each seller posts prices and a submarket is formed by sellers posting the same price. Each buyer directs her search toward a most-attracted submarket in which each buyer is randomly matched with a seller. After the match, a buyer chooses a UoA between the CM-good posting and the money posting. Each agent then receives a lump-sum transfer of money, followed by the matched pair's trade. Upon entering the centralized competitive market, agents trade the CM-good in exchange for money.

3. Equilibrium

We here focus on a stationary monetary equilibrium in which the end-of-period real money balance is constant over time: i.e., $\phi_{t-1}M_{t-1} = \phi_t^h \mu_t^h M_{t-1} = \phi_t^l \mu_t^l M_{t-1}$ where ϕ^i for $i \in \{h, l\}$ is the real price of money in terms of the CM-good when the realized money growth rate is μ^i. Hereinafter we drop the time subscript t and index the next-period (previous period) variable by $+1$ (-1) if there is no risk of confusion.

3.1. Money Demand

In the CM, agents produce, consume the CM-good, and choose the balance of money to be carried into the next-period DM. Let $W(m)$ be the value function pertaining to the beginning of the CM where an agent holds m units of money and let $V(m)$ be the value function pertaining to the beginning of the DM where an agent holds m units of money. Then, the problem for an agent entering the CM with a monetary wealth m is

$$W(m) = \max_{(x,y,m_{+1})} \left[v(x) - y + \beta V_{+1}(m_{+1}) \right] \tag{2}$$

$$\text{s.t. } x + \phi m_{+1} = y + \phi m \tag{3}$$

where m_{+1} denotes the demand for money to be carried into the next-period DM and $V_{+1}(m_{+1})$ is given by

$$V_{+1}(m_{+1}) = \frac{1}{2} \left[V^b_{+1}(m_{+1}) + V^s_{+1}(m_{+1}) \right] \tag{4}$$

with $V^b(m)$ and $V^s(m)$ denoting the value function for a buyer and a seller in the DM, respectively. Substituting y from the constraint (3), we have

$$W(m) = \phi m + \max_{x} \left\{ v(x) - x \right\} + \max_{m_{+1}} \left\{ \beta V_{+1}(m_{+1}) - \phi m_{+1} \right\}. \tag{5}$$

The first order condition for x is $v'(x) = 1$, which implies that all agents consume x^* units of the CM-good such that $x^* = \arg\max[v(x) - x]$ regardless of m. The first order condition for $m_{+1} \in \mathbb{R}_{++}$ is $\phi = \beta V'_{+1}(m_{+1})$, implying that there is no wealth effect and hence all agents exit the CM with an identical balance of money m_{+1}. Therefore, we can conveniently focus on the case where the distribution of money holdings is degenerate at the beginning of each period. Finally, the envelope condition, $W'(m) = \phi$, implies that the value function $W(m)$ is linear as in the typical Lagos-Wright model with a quasi-linear utility function.

3.2. Unit-of-Account Choice

At the beginning of the DM, the preference shock is realized such that a half of agents become sellers and the rest become buyers. Each seller then posts prices in terms of money and the CM-good, respectively. If a buyer chooses the former, money is used as a UoA as well as an MoE, whereas if the latter is chosen, money is used as an MoE only and its role as a UoA is replaced by the CM-good.

Specifically, by posting prices q^m and q^u, a seller commits to produce q^m units of the DM-good in exchange for a unit of money and q^u units of the DM-good per unit of the CM-good. The value function for a seller with m units of money in the DM, $V^s(m)$ in (4), should then satisfy

$$V^s(m) = \max_{(q^m,q^u)} \left\{ \left(\frac{\alpha}{2}\right) \sum_{i\in\{h,l\}} \left[\left(\hat{z}^i - \hat{m}^i q^m\right) + \left(\tilde{z}^i - \tilde{z}^i q^u\right) \right] + W(m+\tau) \right\}. \tag{6}$$

Here $\hat{z}^l = \phi^l \hat{m}^l$, $\hat{z}^h = \phi^h \hat{m}^h$, $\tilde{z}^l = \phi^l \tilde{m}^l$, $\tilde{z}^h = \phi^h \tilde{m}^h$, and $\hat{m}$ ($\tilde{m}$) denotes the nominal amount of transaction when the trade is made according to the price posted in terms of money (CM-good), and τ is a lump-sum transfer of new money such that

$$\tau = \begin{cases} \tau^h = (\mu^h - 1)M_{-1} & \text{if } \mu = \mu^h \\ \tau^l = (\mu^l - 1)M_{-1} & \text{if } \mu = \mu^l. \end{cases} \tag{7}$$

Noting that sellers are identical and competitive, they eventually choose to post the same prices (q^m, q^u). This means that sellers all together form a single submarket. As will be discussed later, a buyer spends all the money she holds and hence, from (6), we have

$$(m+\tau^h)q^m \le (m+\tau^h)\phi^h \tag{8}$$

because otherwise it is cheaper for a seller to acquire money in the following CM and hence she is not willing to trade in the DM. Similarly, we also have

$$(m+\tau^l)q^m \le (m+\tau^l)\phi^l. \tag{9}$$

From (8) and (9), together with $\phi^h < \phi^l$ from the constant real balances in a stationary equilibrium $(m + \tau^h)\phi^h = (m + \tau^l)\phi^l$ and $(m + \tau^h) > (m + \tau^l)$, q^m should satisfy $q^m \leq \phi^h < \phi^l$. Sellers eventually choose q^m such that $q^m = \phi^h$ because the competition among sellers drives equilibrium profit to zero. In addition, (6) implies $\tilde{z}^i \geq \tilde{z}^i q^u$ and hence $q^u \leq 1$. The zero-profit equilibrium condition for sellers also implies $q^u = 1$.

Now, for the posted prices (q^m, q^u), the value function for a buyer with m units of money in the DM, $V^b(m)$ in (4), should satisfy

$$V^b(m) = \max_{(\mathbb{I},\hat{m},\tilde{m})} \left\{ \begin{array}{c} \left(\frac{1-\mathbb{I}}{2}\right) \alpha \sum_{i\in\{h,l\}} \left[u(\hat{m}^i q^m) - \hat{z}^i\right] + \\ \left(\frac{\mathbb{I}}{2}\right) \alpha \sum_{i\in\{h,l\}} \left[u(\tilde{z}^i q^u) - (\tilde{z}^i + \kappa)\right] \end{array} \right\} + W(m + \tau) \tag{10}$$

subject to $\hat{m}^i \leq (m+\tau^i)$ and $\tilde{m}^i \leq (m+\tau^i)$ for $i \in \{h, l\}$. Notice that the UoA decision $\mathbb{I}$ is made before the realization of money growth shock μ, whereas the nominal amount of transaction $(\hat{m}, \tilde{m})$ is chosen after the realization of μ. Since $\mu > \beta$ regardless of the realized μ, there is a strictly positive opportunity cost of holding money and hence an agent is not willing to carry money balances exceeding $\bar{m}$ such that $(\bar{m} + \tau^h)q^m = q^* = \arg\max[u(q) - q]$. Further, noting that there is a single submarket where a measure of sellers is a half, a measure of buyers is also a half because all buyers are willing to visit the single submarket. Therefore, the ratio of sellers to buyers is just one ($\alpha = 1$) so that each buyer is matched with a seller. These together imply that (10) can be simplified as follows:

$$V^b(m) = \max_{\mathbb{I}} \left\{ \left(\frac{1-\mathbb{I}}{2}\right) \sum_{i\in\{h,l\}} u(\boldsymbol{m}^i q^m) + \mathbb{I}\times \left[u(\boldsymbol{z} q^u) - \kappa\right] \right\} + W(0) \tag{11}$$

where $\boldsymbol{m}^h = m + \tau^h$, $\boldsymbol{m}^l = m + \tau^l$ and $\boldsymbol{z} = (m + \tau^l)\phi^l = (m + \tau^h)\phi^h$. It is worth noting that for $\mathbb{I} = 0$ (money posting), the quantity of the DM-good produced relies on the realized inflation via $\boldsymbol{m}^i = m + \tau^i$ for $i \in \{h, l\}$, whereas it is irrelevant for $\mathbb{I} = 1$ (CM-good posting).

Then, from (10) and (11), a buyer's payoff from a bilateral trade for choosing the price quoted in terms of money ($\mathbb{I} = 0$), denoted S_m, is

$$\begin{aligned} S_m &= \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) - \phi^h \boldsymbol{m}^h\right] + \frac{1}{2}\left[u(\boldsymbol{m}^l q^m) - \phi^l \boldsymbol{m}^l\right] \\ &= \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)\right] - \boldsymbol{z} \end{aligned} \tag{12}$$

where the second equality comes from $\phi^h \boldsymbol{m}^h = \phi^l \boldsymbol{m}^l = \boldsymbol{z}$. On the other hand, a buyer's payoff from a bilateral trade for choosing the price quoted in terms of the CM-good ($\mathbb{I} = 1$), denoted S_g, is

$$S_g = u(\boldsymbol{z} q^u) - \boldsymbol{z} - \kappa. \tag{13}$$

Therefore, a buyer chooses the price quoted in terms of the CM-good if and only if $S_m < S_g$ and the CM-good becomes an active UoA. That is, money plays the role of an MoE only and its role as a UoA is replaced by the CM-good.

4. Active Unit of Account

We are now ready to discuss under what conditions money can fulfill the roles of a UoA as well as an MoE. Let Δ denote the relative payoff for a buyer from choosing the price quoted in money, which becomes the following from (12) and (13)

$$\Delta = \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)\right] - \left[u(\boldsymbol{z} q^u) - \kappa\right]. \tag{14}$$

As a preliminary result, the following lemma shows that if there is no inconvenience or disutility cost of converting the CM-good price into money price, buyers choose the price anchored on real value of the CM-good rather than the price anchored on nominal value of money.

Lemma 1 *If* $\kappa = 0$, $\Delta = S_m - S_g < 0$ *and hence buyers always prefer the price quoted in the CM-good to the price quoted in money.*

Proof. We need to show that

$$\Delta|_{\kappa=0} = \frac{1}{2}\left[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)\right] - \left[u(\boldsymbol{z} q^u)\right] < 0.$$

Since $q^u = 1$ and $q^m = \phi^h$, $u(\boldsymbol{z} q^u) = u(\boldsymbol{z})$ and $u(\boldsymbol{m}^h q^m) = u(\boldsymbol{m}^h \phi^h) = u(\boldsymbol{z})$. Furthermore, since $\boldsymbol{z} = \phi^l \boldsymbol{m}^l > \phi^h \boldsymbol{m}^l$ due to $\phi^h < \phi^l$, we have $u(\boldsymbol{m}^l q^m) = u(\boldsymbol{m}^l \phi^h) < u(\boldsymbol{m}^l \phi^l) = u(\boldsymbol{z})$. These together immediately imply that $\Delta|_{\kappa=0} < 0$. ■

Intuitively, in the absence of any cost of converting the CM-good price into money price ($\kappa = 0$), a buyer can secure a stable consumption against inflation uncertainty by choosing the price quoted in terms of the CM-good. But if a buyer chooses the price quoted in money that is not anchored on real value, her consumption varies depending on the realized inflation. Hence, an active UoA becomes the CM-good if $\kappa = 0$.

This result implies that if there is some inconvenience in converting the CM-good price into money price (i.e., $\kappa > 0$), a nominal-value anchoring economy can arise where money becomes an active UoA as well as an MoE.

Proposition 1 *There exists* $\bar{\kappa} \in \mathbb{R}_{++}$ *such that* $\Delta|_{\kappa=\bar{\kappa}} = 0$. *For* $\kappa > \bar{\kappa}$, *buyers choose money as an active unit of account.*

Proof. Notice that the first bracket in the right-hand side of (14) is irrelevant to κ, whereas the second one decreases as κ becomes larger. This together with Lemma 1, $\Delta|_{\kappa=0} < 0$, implies that there is $\bar{\kappa} > 0$ satisfying $\Delta|_{\kappa=\bar{\kappa}} = 0$ and $\Delta|_{\kappa>\bar{\kappa}} > 0$. ■

Proposition 1 simply states that if the cost associated with converting an imaginary UoA into tangible fiat-money units is sufficiently high, money becomes both an MoE and a UoA. Along this line, Fisher (1913) argued against the separation of a UoA from an MoE:

> Not only would the multiple standard necessitate much laborious calculation in translating from the medium of exchange into the standard of deferred payments, and back again but, if, as has been suggested, the employment of a multiple standard were at first optional, the result would be that many business men whose prosperity depended on a narrow margin between their expenses and receipts would be injured rather than benefited by having one side of their accounts predominantly in the actual dollar and the other in the ideal unit.

The threshold value of the conversion cost, $\bar{\kappa}$, can be interpreted as a risk premium in the sense that it captures the maximum disutility cost buyers are willing to bear for ensuring a stable consumption against inflation uncertainty. If the cost of converting the CM-good price (i.e., an imaginary UoA) into money price (i.e., an MoE price) exceeds this upper bound, then agents are not willing to hedge the consumption volatility against inflation uncertainty because the cost to do so is greater than its benefit.

Now, what are the critical factors that determine the magnitude of the threshold $\bar{\kappa}$? The following proposition suggests that the volatility of inflation does matter for $\bar{\kappa}$ and, in particular, a higher volatility of inflation implies a higher $\bar{\kappa}$.

Proposition 2 *If there is no uncertainty in inflation ($\varepsilon = 0$), then $\bar{\kappa} = 0$ and hence money is always an active unit of account for $\kappa > 0$. In addition, $\bar{\kappa}$ increases with the uncertainty in inflation.*

Proof. For the first claim, if $\varepsilon = 0$, $\mu^h = \mu^l = \bar{\mu}$, $\boldsymbol{m}^h = \boldsymbol{m}^l = \boldsymbol{m}$, and $\phi^h = \phi^l = \phi$. Therefore $\Delta = (1/2)[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)] - [u(\boldsymbol{z} q^u) - \kappa] = (1/2)[u(\boldsymbol{m}\phi) +$

$u(\boldsymbol{m}\phi)] - [u(\boldsymbol{z}) - \kappa] = (1/2)[u(\boldsymbol{z}) + u(\boldsymbol{z})] - [u(\boldsymbol{z}) - \kappa] = \kappa$. Then the definition of $\bar{\kappa}$ ($\Delta|_{\kappa=\bar{\kappa}} = 0$) implies $\bar{\kappa} = 0$ for $\varepsilon = 0$. For the second claim with $\varepsilon > 0$, since $\boldsymbol{m}^h = \mu^h M_{-1} = \bar{\mu}(1+\varepsilon)M_{-1}$ and $\boldsymbol{m}^l = \mu^l M_{-1} = \bar{\mu}(1-\varepsilon)M_{-1}$ due to money-market clearing condition, $\{\partial \left[u(\boldsymbol{m}^h q^m) + u(\boldsymbol{m}^l q^m)\right] / \partial\varepsilon\} < 0$. This, together with $\{\partial \left[u(\boldsymbol{z} q^u) - \kappa\right] / \partial\varepsilon\} = \{\partial \left[u(\boldsymbol{z}) - \kappa\right] / \partial\varepsilon\} = 0$ implies that $(\partial\Delta/\partial\varepsilon) < 0$. Therefore as ε increases, $\bar{\kappa}$ that make $\Delta = 0$ should increase. ■

Intuitively, if there is no uncertainty in inflation, money is an active UoA as well as an MoE even if the conversion cost of an imaginary UoA into money is sufficiently small. However, as the volatility of inflation (ε) increases for a given conversion cost $\kappa > 0$, the threshold level $\bar{\kappa}$ increases and the relative payoff from choosing money as a UoA, Δ in (14), eventually becomes negative for a sufficiently high ε. That is, for a sufficiently high uncertainty in inflation, the CM-good becomes an active UoA and an MoE is no longer equivalent to a UoA. Therefore, even though money is ideal as an MoE in the sense that it is divisible, durable, recognizable and portable, it can hardly function as a UoA if its value is too unstable. It is the volatility of inflation rather than the level of inflation itself that determines whether an MoE becomes an active UoA. Finally, despite the importance of portability as an MoE during a hyperinflation, it is irrelevant to the choice of a UoA. This is because trade is eventually cleared by transferring money as an MoE regardless of the choice of a UoA–money or the CM-good.

This finding is consistent with Keynes (1923), Doepke and Schneider (2013), and Kim and Lee (2013) who claim that as the volatility of an MoE increases, its quality as a UoA is deteriorated and an alternative UoA would emerge. That is, if the value of money as an MoE is relatively unstable, it will not be appropriate to adopt money as a yardstick to represent the terms of trade. An alternative object anchored on real value will take up the role of a UoA even in the economy where money is used as an MoE. This provides an explanation for why some countries adopted a UoA different from an MoE during hyperinflations at some inconvenience. For instance, during the German hyperinflation in the early 1920s, prices were typically quoted in gold Marks, while payments were made by paper Marks (fiat money) (Wolf 2002). Similar instances were recently observed in some Latin American countries such as Brazil, Chile, Colombia, Ecuador, Mexico, and Uruguay which introduced a separate UoA other than the domestic currency in the aftermath of high inflation (Shiller 2002).

5. Nominal Rigidity and Non-neutrality of Money

In the macroeconomic profession including new monetarists such as Williamson and Wright (2010), there seems to be a wide agreement that the price level is somewhat sticky in the real world. An equilibrium with money as an active UoA has a contrasting implication for the nominal rigidity compared to an equilibrium with the CM-good as an active UoA. Let p_m and p_g denote respectively the equilibrium price level (i.e., the units of money exchanged for the quantity of the DM-good) when money and the CM-good are active UoA. Then p_m and p_g become as follows respectively:

$$p_m = \begin{cases} \frac{\boldsymbol{m}^h}{\boldsymbol{m}^h q^m} = \frac{1}{q^m} = \frac{1}{\phi^h} & \text{if } \mu = \mu^h \\ \frac{\boldsymbol{m}^l}{\boldsymbol{m}^l q^m} = \frac{1}{q^m} = \frac{1}{\phi^h} & \text{if } \mu = \mu^l. \end{cases}$$

$$p_g = \begin{cases} \frac{\boldsymbol{m}^h}{\boldsymbol{z} q^u} = \frac{1}{\phi^h q^u} = \frac{1}{\phi^h} & \text{if } \mu = \mu^h \\ \frac{\boldsymbol{m}^l}{\boldsymbol{z} q^u} = \frac{1}{\phi^l q^u} = \frac{1}{\phi^l} & \text{if } \mu = \mu^l \end{cases}$$

where we use $q^m = \phi^h$ and $q^u = 1$ in equilibrium.

When the CM-good is an active UoA which is different from an MoE (i.e., money), $1/\phi^h > 1/\phi^l$ due to $\phi^h < \phi^l$ in equilibrium. That is, the price level varies with the growth rate of money supply so that the price level is higher (lower) when the growth rate of money supply is high (low). This implies the flexible price level when the CM-good is an active UoA in equilibrium.

On the other hand, when money is an active UoA as well as an MoE, the equilibrium price level is given by $1/\phi^h$ regardless of the money growth rate: i.e., the price level is not adjusted immediately to the change in the money growth rate.[4] This implies the rigid or sticky price level in the short run when money is an active UoA in equilibrium.

It is worth noting that the choice of a UoA in the presence of money as an MoE determines endogenously the nominal rigidity or flexibility. An economy adopting money as a UoA yields the short-run nominal price rigidity, whereas an economy adopting an object that is anchored on real value as a UoA yields the nominal price flexibility. This has an important testable implication for the nominal rigidity in the real world: the more (fewer) firms post prices in terms of a UoA different from money, the more (less) flexible the price level becomes.

Further, noting that the nominal rigidity is one of the most plausible explanations for the non-neutrality of money in the short run, the choice of a UoA also affects the relationship between the money growth rate and output. That is, when the money is an active UoA, the quantity of the DM-good produced is positively correlated with the growth rate of money: i.e., $m^h q^m > m^l q^m$. In most modern economies where money fulfills the roles of a UoA as well as an MoE, this implies a traditional short-run Phillips curve which represents a positive relation between money growth and output. However, when the CM-good is an active UoA, the quantity of the DM-good produced is irrelevant to the realized money growth rate: i.e., $\phi^h \boldsymbol{m}^h = \phi^l \boldsymbol{m}^l = \boldsymbol{z}$. This is consistent

[4] As a related study, Jovanovic and Ueda (1997) show that if the parties to a contract expect that the price level will be measured with delay, they will not write a fully indexed contract because they know that they would choose to renegotiate it.

with Shiller (2002) who claims that if an MoE is separated from a UoA that is indexed to consumer price index, the effects of sticky prices on the macroeconomy would be substantially lessened.

6. Concluding Remarks

Monetary economics so far has focused almost exclusively on money as an MoE as in Jevons (1875), Kiyotaki and Wright (1989), and Kocherlakota (1998). The role of a UoA has stayed out of the spotlight and there seems to be a tendency to equate an MoE with a UoA. However, some real-world observations suggest that it would not be the case, particularly during the hyperinflations in Germany and in some Latin American countries.

In this paper, we show that such a separation arises if there is considerable uncertainty in inflation. That is, if the value of money is too unstable, an alternative object anchored on real value replaces money as an active UoA. This implies that, in order to fulfill the roles of money as a UoA as well as an MoE, stability is required other than the properties advocated by Jevons (1875): i.e., money should be divisible, durable, recognizable, portable and stable.

Finally, the choice of a UoA in the presence of fiat money as an MoE determines endogenously the nominal price rigidity or flexibility. In an economy adopting money as a UoA as well as an MoE, the price level is sticky in the sense that it is not adjusted immediately with the growth rate of money supply. However, an economy adopting an object anchored on real value as a UoA yields the nominal price flexibility. In Chile where prices of some goods are posted in *Unidad de Fomento* (CPI-Indexed-UoA) while prices of other goods are posted in Peso (MoE), it would be interesting to see whether there is any difference in the price rigidities between the two groups of goods.

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